Name: ___________________________________ Date: ______________

Algebra 2H: Unit 10 – Logarithmic Functions Unit Exam

Part I: Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answer choice in the space provided.

________________1. If $\log_x 2 = \frac{1}{3}$, then $x =$

(1) $2\frac{1}{3}$

(2) $\sqrt[3]{2}$

(3) 8

(4) 4

________________2. The graph of the equation $y = \log_4 x$ lies entirely in which quadrants?

(1) I & IV

(2) I & II

(3) III & IV

(4) II & III

________________3. For which value of x is $f(x) = \ln x$ undefined?

(1) 0

(2) $\frac{\pi}{4}$

(3) $\frac{1}{3}$

(4) e

________________4. What is the *exact* value of t that satisfies $50 = \frac{100}{1+4e^{-0.1t}}$?

(1) $t = -10\ln(4)$

(2) $t = 10\ln(4)$

(3) $t = \frac{5}{2}$

(4) $t = -\frac{5}{2e}$

We should have skipped this one, too, so don't be alarmed if you missed it! In reality, it's a lot like solving a literal equation, where you're trying to isolate the variable *t*.

________________5. To the nearest *hundredth*, the value of x that solves $5^{x-4} = 275$ is

(1) 6.73 (3) 8.17
(2) 5.74 (4) 7.49

________________6. $\log a + \frac{1}{2}\log b =$

(1) $\log\sqrt{ab}$ (3) $\log(a+\sqrt{b})$
(2) $\log a\sqrt{b}$ (4) $(\log a)(\frac{1}{2}\log b)$

________________7. Which of the following is equivalent to $\log\frac{\sqrt{r}}{s}$?

(1) $\frac{2\log r}{\log s}$ (3) $\frac{1}{2}\log r - \log s$
(2) $2\log r - \log s$ (4) $\frac{\log r - \log s}{2}$

________________8. The log form of $y = a^x$ is

(1) $y = \log_a x$ (3) $a = \log_x y$
(2) $x = \log_a y$ (4) $x = \log_y a$

________________9. $\log\frac{a^4}{1000}$ can be rewritten as

(1) $4\log a - 3$ (3) $4a - 1000$
(2) $\frac{4\log a}{3}$ (4) $\frac{4a}{1000}$

________________10. If log 7 = a, then log 490 equals

(1) $2a + 10$
(2) $10a^2$
(3) $a + 70$
(4) $2a + 1$

________________11. Which of the following represents the value of $\ln\left(\sqrt[a]{e^b}\right)$?

(1) $\sqrt{\frac{b}{a}}$
(2) $\frac{b}{a}$
(3) $\frac{a}{b}$
(4) $e^{\frac{b}{a}}$

________________12. The inverse of $y = 5^x$ is obtained by reflecting $y = 5^x$ in the line

(1) $y = x$
(2) y-axis
(3) x-axis
(4) $y = -x$

<u>Part II</u>: Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a numerical answer with no work shown will receive only 1 credit.

13. Solve for *x*, ***to the nearest hundredth***. Only an algebraic solution will receive full credit.

$5^{2x} + 9 = 40$

<u>- 9 -9</u> Start to isolate the variable.

$5^{2x} = 31$

$\log(5^{2x}) = \log 31$ Take the log of both sides.

$2x(\log 5) = \log 31$ Apply the power rule.

$x = 1.07$ Use calc to divide by log 5, then by 2. Round.

14. Solve for the *exact* value of *t* that satisfies the equation $15 = \dfrac{30e^{0.4t}}{e^{0.4t} + 5}$. Only an algebraic solution is allowed.

Skip this one. For anyone who did try it, here's what the answer would have been:

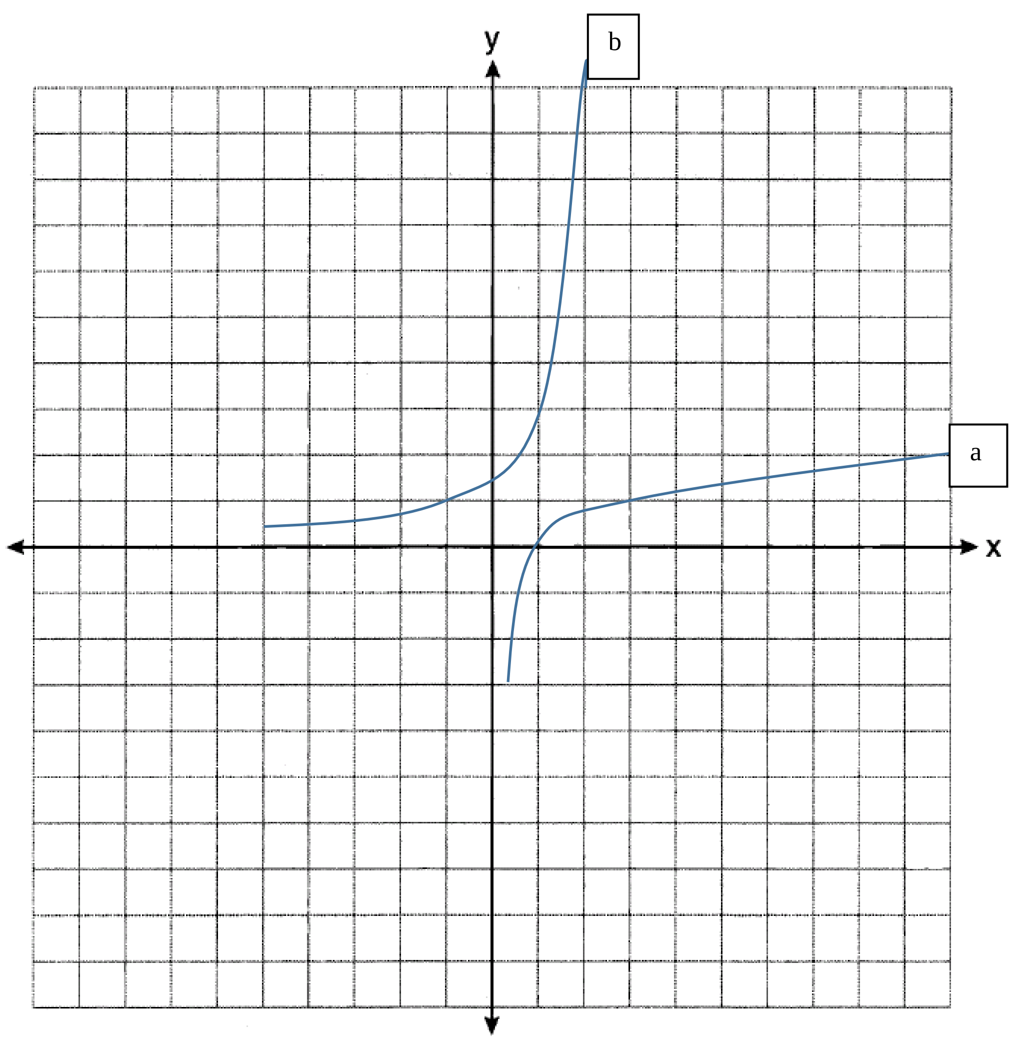
$t = \dfrac{\ln 5}{.4}$

<u>Part III</u>: Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a numerical answer with no work shown will receive only 1 credit.

15. (a) Graph and <u>label</u> the function $y = \log_3 x$.

(b) On the same set of axes, graph the inverse of the function in part **a**.

(c) What is the equation of the graph in part **b**? ________ $y = 3^x$____________________



16. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$T = T_a + (T_0 - T_a)e^{-kt}$	T_a = the temperature surrounding the object
	T_o = the initial temperature of the object
	t = the time in hours
	T = the temperature of the object after t hours
	k = decay constant

The turkey reaches the temperature of approximately 100° F after 2 hours. Find the value of k, to the *nearest thousandth*. Show how you arrived at your answer.

$100 = 325 + (68 – 325)e^{-k(2)}$	Set up equation with numbers from problem
$100 = 325 + (-257)e^{-2k}$	Simplify in ( )
$-225 = -257e^{-2k}$	Begin solving for k.
$-225/-257 = e^{-2k}$	
ln (225/257) = -2k (ln e)	Take the natural log (since we're using e)
k = .066	Use calc and divide by -2. Round.

Using your value of k, determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m. Show how you arrived at your answer.

Notice that this is **7 hours later**.

$T = 325 + (-257)e^{-.066(7)}$

T = 163 degrees

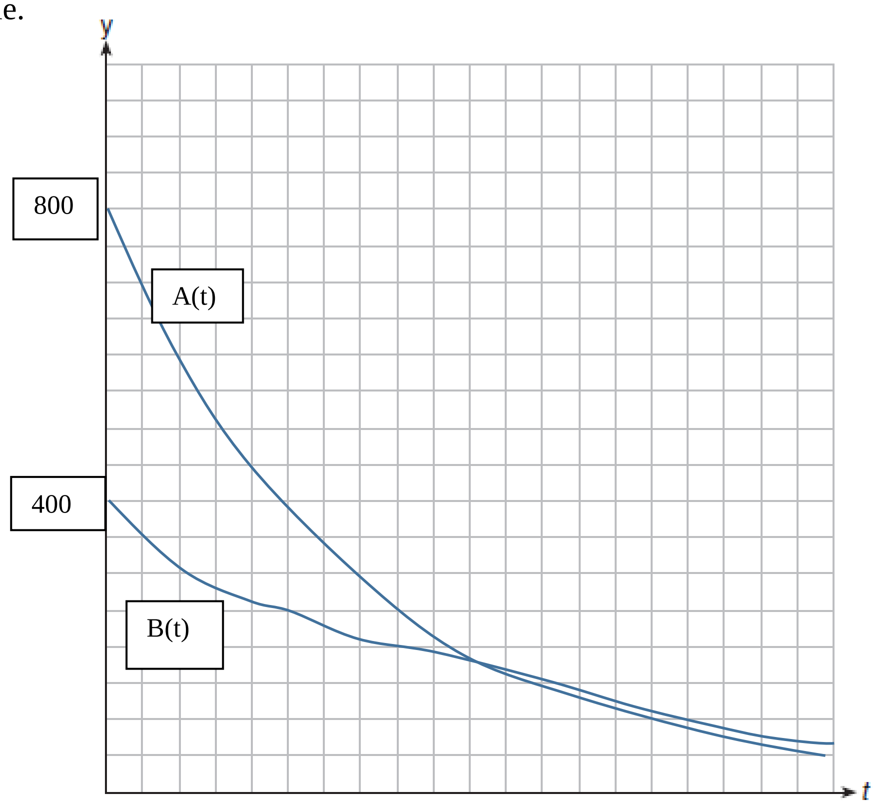
<u>Part IV</u>: Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a numerical answer with no work shown will receive only 1 credit.

17. Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_o(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_o is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$A(t) = 800e^{-.347t}$ $B(t) = 400e^{-.231t}$

Graph and label each function on the set of axes below. Be sure to include an appropriate scale.



I apologize for the crudeness of the drawing…I did my best!

CONTINUE→

Using your graph on the previous page and/or your graphing calculator, find the time, to the nearest hundredth, where the amount of the drug left in the patients' body is the same.

5.98 hours

Determine whether patient A or B is the first to have 25 milligrams or less of the drug in their system. Justify your answer. *Only an algebraic solution will receive full credit*.

$800e^{-.347t} = 25$	$400e^{-.231t} = 25$	Set up both equations.
$e^{-.347t} = 1/32$	$e^{-.231t} = 1/16$	Isolate the e.
$-.347t (\ln e) = \ln (1/32)$	$-.231t (\ln e) = \ln (1/16)$	Take the ln of both
$A (t) = 9.99$ hours	$B(t) = 12.00$ hours	Calculator time!

Therefore, patient A is the first one!